Phonological Complexity is Subregular: Evidence from Sign Language

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This paper brings recent results on the computational complexity of natural language to bear on the question of modality. Specifically, I address whether the computational properties of phonology hold across the articulatory systems of speech and sign. A theory-neutral approach can highlight modality similarities and differences and allow for insightful comparison. This paper discusses the sequential nature of sign phonology, and overviews the recent computational background characterizing a class of formal transductions. I apply this to several sign processes — contact metathesis, final-syllable reduplication, and compound reduction — and compare them to their spoken equivalents. These analyses suggest an amodal algebraic phonology independent of modality, and allow for promising new means to analyze issues of linguistic modality and the cognitive status of phonological knowledge.

1 Introduction

Research into the nature of the phonological system and its status as a cognitive module has wrestled with the relationship between posited innate phonological universals and their phonetic realization. (Berent 2013) discusses the range this relationship has, from phonetics subsuming the phonological grammar to an emergent “grounded phonology” alternative (that an autonomous phonological grammar is only grounded in phonetics), to the possibility that the emergence of the phonological grammar in ontogeny is unaffected by phonetic substance. This debate is important because to the extent phonological capacity is not innate, it must therefore be learned by mechanisms that use experience as evidence (e.g., statistical learning, analogical reasoning, and the like).

Berent acknowledges that grammatical universals are grounded in phonetics, and that phonetics might well play a critical role in the development of the phonological grammar in both ontogeny and phylogeny, but presents several arguments towards an "algebraic" view of phonology distinct from articulatory considerations. She first supports this claim by highlighting that "some phonological constraints are phonetically unmotivated, whereas others are amodal – shared by both spoken and signed languages."

Until recently, signs were widely assumed to be iconic wholes. William Stokoe’s landmark work demonstrated systematically that there is indeed a level of sign language structure that corresponds to phonology (Stokoe 1960, Stokoe et al. 1965). Stokoe showed that sign languages have duality of patterning: a meaningful level of structure, as well as a level that is made up of a list of meaningless, yet linguistically contrastive elements, the number of which is finite and reasonably small. This discovery tells us that the human brain organizes the language transmission system in a particular way, even where the physical means of transmission is radically different from that of languages in the more widespread modality. It suggests that defining phonology in terms of sound patterns is too narrow and instead "phonology is the level of linguistic structure that organizes the medium through which language is transmitted" (Sandler & Lillo-Martin 2006).

Berent then showcases the "distinct computational properties of phonetics and phonology". While these claims are convincing, they and many arguments in this issue are heavily theory-dependent, so that claims about modality or computation are often defined within theories that were not built to handle them. Often, this muddies the issue and prevents real understanding or translation to other domains. In order to examine amodal computation more neutrally, we can look at it in terms of their computational complexity.

Classifying natural language patterns in terms of their computational complexity—defined in this paper as the amount of computational power needed to recognize and/or generate the pattern—is a theory-neutral approach to understanding what kinds of patterns can and cannot exist in natural language. In addition, computational analyses of patterns in different linguistic domains offer a neutral perspective on how these domains fundamentally differ. Additionally, these classes have well-defined cognitive parallels that suggest exactly the minimum information a cognitive mechanism needs to be mindful of in order to correctly compute whether strings belong to the language (Rogers & Pullum 2011; Rogers *et al.* 2013).

This paper will proceed in the following way: section 2 outlines recent results on formal language complexity and its application to phonological processes. Section 3 expands on the issue of representation in signs and Section 4 overviews the complexity of several sign processes. Section 5 connects this analysis to issues of phonological computation and cognition.

2 Formal Complexity and Phonological Processes

2.1 The Subregular Hypothesis

The computational power needed to generate natural language patterns is classified by particular regions of the Chomsky Hierarchy of formal languages. Previous work has shown that syntactic string patterns exist which inhabit the context-free and context-sensitive regions (Chomsky 1956; Shieber 1985; Kobele 2006), while virtually all phonological patterns are known to be regular (i.e., finite state) (Johnson 1972; Koskenniemi 1983; Kaplan & Kay 1994). However, the regular class generates patterns unseen in natural phonology, and substantial recent evidence situates phonotactic patterns distinctly to classes in the ***subregular hierarchy*** (see Fig. 1). Pretty much all segmental phonotactics can be modeled with NonCounting languages (Graf 2010), and a huge majority with Strictly Local and Strictly Piecewise ones (see (Heinz 2018) for an overview).

Another important contribution of formal complexity research in phonology is that phonological Underlying Form - Surface Form *mappings* can be represented by proper subclasses of the regular *relations*, which are characterized by *Finite-State Transducers* (FST). However, the subregular hierarchy cannot be used directly to study transductions, which are sets of string pairs, not stringsets, and the hierarchy

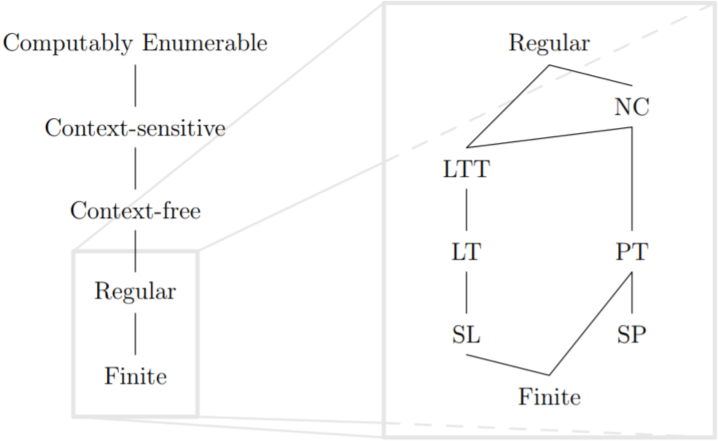


Figure 1: Chomsky Hierarchy of Languages and the Subregular Hierarchy of Languages. Lines indicate proper inclusion from top to bottom. NC = NonCounting, LTT = Locally Threshold Testable, LT = Locally Testable, SL = Strictly Local, PT = Piecewise Testable, SP = Strictly Piecewise. (from (Chandlee & Heinz 2018))

only describes formal *languages*. The ***subregular mapping hierarchy***, shown in Figure 2 (Mohri 1997; Chandlee 2014) has led to several key results for phonology, though the alignment between subregular mappings and transductions is not completely known.

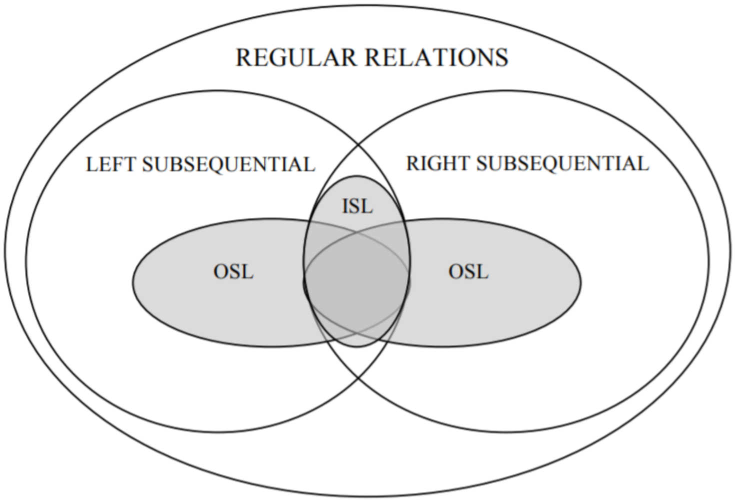


Figure 2: Relationships among Subregular Mappings: left/right Output Strictly Local (OSL), Input Strictly Local (ISL), subsequential, and regular (from (Chandlee 2017))

Chandlee (2014), Chandlee *et al.* (2015), and Chandlee & Heinz (2018) show that phonological maps that correspond to local processes (where the target and triggering context form a contiguous substring of bounded length) can all be classified in one of the SL regions of the maps hierarchy. In the finite state formalism, this means these types of phonological generalizations can be described with FSTs that have the characteristic properties of the Left/Right OSL, and/or ISL classes. Importantly, while these relationships will be defined here in terms of FSTs, the complexity hierarchy they define is *independent* of their representation with FSTs, and they hold regardless of the formalism describing them. This paper makes heavy use of the ISL function class, so I will now illustrate it intuitively and with an example.

2.2 Input Strictly Local Functions

Imagine you are a court stenographer asked to read a string of symbols in order from the beginning and write the English output. As you begin reading the code (from either direction) you begin writing the output (in the same direction). The Strict Locality in ISL functions comes from having some integer k such that your output y at any given point in this process only depends on the last k symbols you read in the input x. Figure 3 illustrates this point with $k = 2$. This is a very informal definition; readers are referred to Chandlee (2014) for additional mathematical details of the definition and proofs.

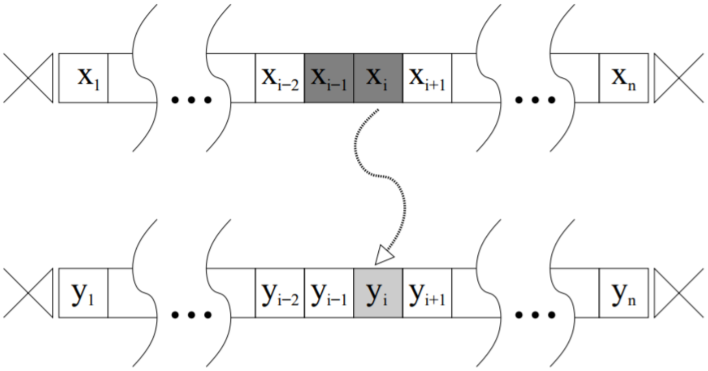


Figure 3: The Markov nature of an Input Strictly 2-Local function, where the output string yi of each input element xi depends only on xi and $xi - 1$. (from Chandlee & Heinz (2018))

As an example, consider a function for word-final devoicing over the input string tad. Given this input, the ISL FST in Figure 4 begins in state 0 and loops back to state 0 on the first two input symbols t and a, outputting the input unchanged. When it next reads in d, it has a choice. It can either move to state 2 and output t, or it can move to state 1 and output d. For the map $tad \rightarrow tat$ to be valid, there must be at least one path through the FST that reads tad, outputs tat, and ends in a final state. For the derivation in Figure 4 the top row shows the input symbols, the bottom row the corresponding output, and the center row the series of visited states.

To reiterate, a phonological map is ISL provided that the output that is written at any given point only depends on the current input symbol and the preceding $k - 1$ symbols of the input string. Compare the ISL final devoicing example above to a hypothetical (and typologically unattested) version in which only consonants following an even number of vowels: $f(taad) = taat$, and $f(taaad) = taaad$. This function is not ISL for any k (i.e., no matter what value we assign to k, the

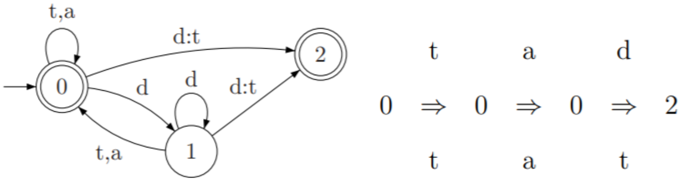


Figure 4: Left: FST for final devoicing. Right: sample derivation for the input 'tad' (from Chandlee & Heinz (2018))

defining property of ISL functions will fail to hold).

3 Sequential and Simultaneous Models of Sign Phonology

Analyzing sign at any linguistic representational level is problematic since most definitions and terminology are borrowed from spoken linguistics. It is generally accepted to equate the concept "sign" with "word" from a morphosyntactic point of view, though the relation between morpheme, syllable, and segment is not a direct isomorphism. Whether signed syllables even exist is a contentious enough issue for Sandler & Lillo-Martin (2006) to devote an entire chapter to the subject. Wilbur (2011) provides a comprehensive overview of the topic of sign syllables, arguing convincingly that by any measure, defining any canonical traits of a sign syllable is a slippery matter.

While the true nature of a sign syllable may remain contentious, various models of syllables can highlight particular characteristics in it. Though a model may be flawed in some respects, adopting its perspective can lead to important insights into the nature of the syllable. Advances in autosegmental phonology and morphology provided a conceptual and formal framework for exploring the relationship among sequentiality, segmentation, and simultaneity in sign languages, a framework first tapped for sign language in the Move–Hold model of Liddell & Johnson (1989). This approach led to the Hand Tier model of sign language structure (Sandler 1986, 1989). In this representation L stands for location and M for movement. HC represents the hand configuration, which has its own complex structure that I will not discuss here.

The Hand Tier model makes use of parts of Stokoe's and Move–Hold characterizations, while rejecting others. In the Hand Tier model, sequential segmental structure established is maintained, some phonetic features, and sequential Movement There are many differences, though. Locations replace the holds of the Move–Hold model as a major segmental category. For Sandler, Lengthened holding of the hand or hands at some location is related to prosody or morphological structure, rather than some underlying category of signs.

Another distinguishing feature of the Hand Tier model is the representation of Stokoe's idea of hand configuration, location, and movement as major category types, though not simultaneously organized with respect to each other. In Sandler's model, locations and movements are organized in a sequence, while hand configuration typically characterizes the whole sequence simultaneously. Thus Sandler manages to captures both sequential and simultaneous aspects of sign language structure. Major categories do not contain "phonemes" per se but, like the major phonological categories of spoken languages, are comprised of subclasses of features.

Locations are the start and end point that the hand traverses in articulating the sign. In the citation form of the sign 'IDEA', for example, pictured in Figure 1, the first location is in contact with the head of the signer, and the second location is a slight distance in front of the first. The major body area, or place of articulation, is the head, and that place characterizes the whole sign,

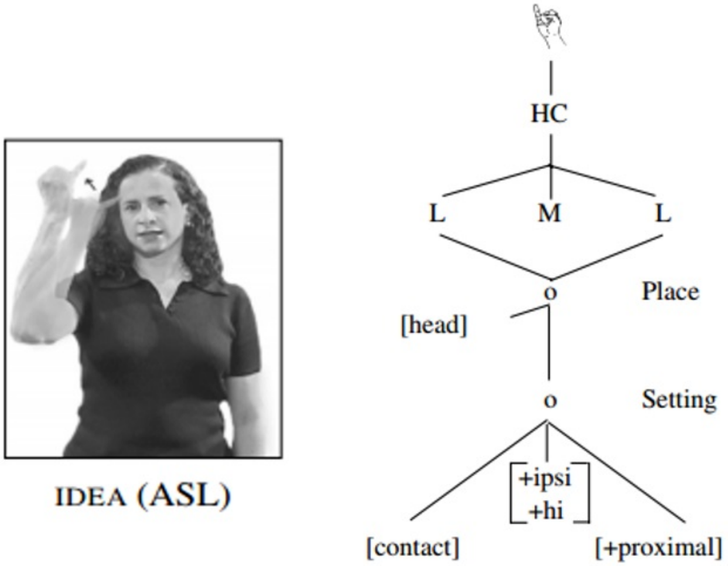


Figure 5: Citation form and Hand Tier representation of ASL 'IDEA' (from (Sandler & Lillo-Martin 2006)).

The Hand Tier model posits that the start and end locations are separately and sequentially represented. The hand moves from one location to another in relation to the place of articulation. Thus, Sandler avoids the Liddell-Johnson hold-deletion rules, but assumes that movement epenthesis is a phonetic effect not needing a phonological rule, as they claim. The hand or hands usually move from one location to another, but generally, sign morphemes are characterized by just one handshape. Here I follow Sandler's representation, so the diagram in Fig. 5 substitutes a handshape icon for representation of what is a complex category.

Sandler's Hand Tier model attempts to eliminate the representational redundancy while capturing generalizations about sign form. Crucially, only changing features are represented sequentially.The others are multiply associated. Hand configuration and place are always multiply associated, reflecting the fact that they are instantiated simultaneously across segments. Thus, in Fig 5, The first location has the feature [contact], meaning that the hand is in contact with the head; and the second has the feature [proximal], meaning the hand is a short distance from the head.

Sandler represents hand configuration as a single, multiply associated category in the Hand Tier model. This is motivated by the fact that most signs are characterized by a single hand configuration which does not vary throughout the sign. However, as Liddell and Johnson note, many signs are characterized by some change in the handshape, which they represent in separate segmental cells.

The picture that emerges from Sandler is one in which signs have a very constrained and predictable sequential structure. However, in this representation, much of the phonological material of signs co-occurs simultaneously. Sandler's Hand Tier model is explicity designed to capture and tease apart these generalizations via grouping features into categories, and one-to-many autosegmental associations of these categories. I now turn to an analysis of some processes characterized by this representation

4 Strictly Local Mappings in Sign Phonology

Now that the intuition behind subregular mappings, and in particular ISL functions, has been laid down, this section will analyze the complexity of several mappings in sign language: contact metathesis, final syllable reduplication, and compound reduction. These processes were chosen for several reasons. First, they exhibit a range of local and non-local behavior. Second, their domain of application ranges from the segmental level to the syllabic level. Third, they range from the strictly phonological (in the case of metathesis) to the arguably morphophonological (compound reduction, reduplication). For spatial considerations, when FSTs are shown I will not provide a sample derivation.

A careful reader may notice that I am using the term 'sign language' when all my examples come from ASL. The choice of language was determined by technical and spatial convenience and is therefore arbitrary, and the illustrations are put forward as examples of general phonological properties of sign languages.

4.1 Contact Metathesis

One sign process making crucial reference to discrete locations is metathesis, which switches the first and last locations of a sign (Liddell & Johnson 1989). In signs in which the signing hand makes contact at two different settings within one major body area (such as the head or chest), the order of the two may be reversed, as shown in Figure 6. In 6a, DEAF follows FATHER. FATHER is signed at the forehead and the first location for DEAF is at the cheek, followed by the second location, at the chin. In 6b, DEAF follows MOTHER, a sign made at the chin. Influenced by this lower location, the sign DEAF begins with the chin location in this context, and ends at the cheek. Liddell and Johnson claim that the conditioning environment for metathesis is the location of the preceding sign.

In order to characterize such a process, in (1) I represent each sign as a string of L and M segments. Each L segment is indexed indicating its unique location. Each sign is augmented with word edge markers. Metathesis switches the indexed L segments in the second sign (3 and 4).

(1) $\rtimes L_1 M L_2 L_3 M L_4 \ltimes \rightarrow \rtimes L_1 M L_2 L_4 M L_3 \ltimes$

Because of the intervening Movement segment within the metathesized syllable, this is a particular type of non-local metathesis where the intervening segment is

a. FATHER DEAF (ASL) b. MOTHER DEAF (ASL)

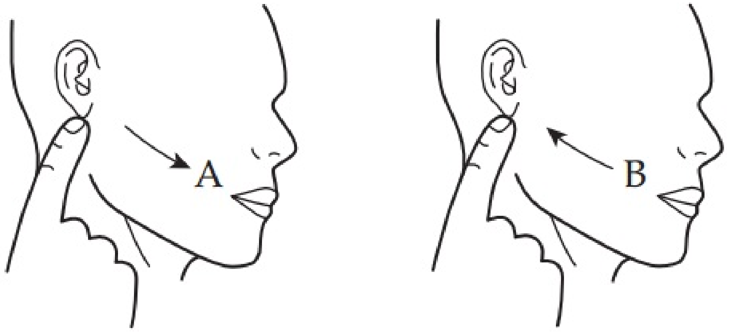


Figure 6: Contact Metathesis in ASL 'DEAF' (adapted from (Sandler & Lillo-Martin 2006; Wilbur 2011))

bounded. This type of metathesis is widely present in spoken languages. Chandlee (2014) cites an example from Cuzco Quechua (Davidson 1977), where a liquid and glide segment exchange positions over an intervening vowel:

(2) ruyaq → yuraq

Chandlee's analysis of these types of patterns decomposes metathesis into component copying and deletion processes, with the requirement that the inserted string to be a substring of either the left or right context of the insertion point. Both metathesized segments are independently copied and then the originals deleted, yielding metathesis in the final output. For the ASL DEAF example, these rewrite rules would correspond to the following:

(3) $\emptyset \rightarrow L_3 \setminus L_3 M\_$

(4) $L_3 \rightarrow \emptyset \setminus \_M L_3$

(5) $\emptyset \rightarrow L_2 \setminus \_M L_3 L_2$

(6) $L_3 \rightarrow \emptyset \setminus L_4 M L_3\ \_$

Since each operation has the form of an SPE-style rule$A \rightarrow B \setminus C_D$, the corresponding FST is ISL. What is crucial to ASL metathesis being SL, however, is that the amount of material that intervenes between the two metathesizing segments is bounded. In this case, the intervening material is a single Movement, so it is bounded by length 1. This bound allows us to identify the k-value of the SL function, which for the rules (3-6), is either 3 or 4.

4.2 Final-Syllable Reduplication

Another process that makes reference to the syllable is morphological reduplication. Many of the temporal aspects described for ASL (Klima & Bellugi 1979), in particular those expressing duration or iteration, involve total reduplication of the base. However, when ASL compounds are reduplicated, the reduplicated element is the final syllable (Sandler 1987b, 1989). If the compound is reduced and monosyllabic, like MIND+DROP=FAINT (see Figure 9), then the whole form is reduplicated, shown in various forms in Fig. 7.

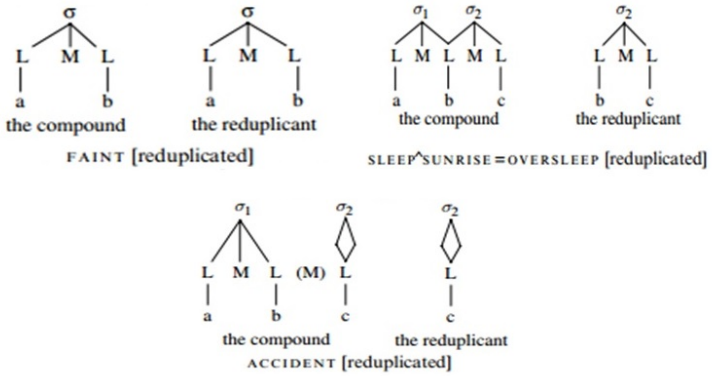


Figure 7: ASL Final Syllable Reduplication (from (Sandler & Lillo-Martin 2006))

Again, the key feature of this reduplication process is its bounded nature. Since the reduplication is contained to single syllables, and since the syllable length in sign is maximally bounded, we can guarantee that there is some finite k-value for the SL function, in this case 4. With a fixed k-value and a fixed alphabet of L,M, we can build a FST for ASL partial reduplication as noted in Figure 8.

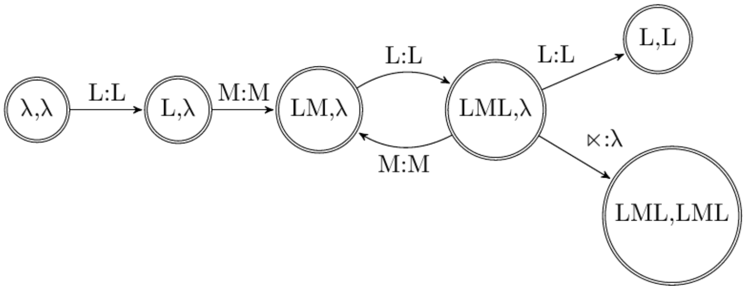


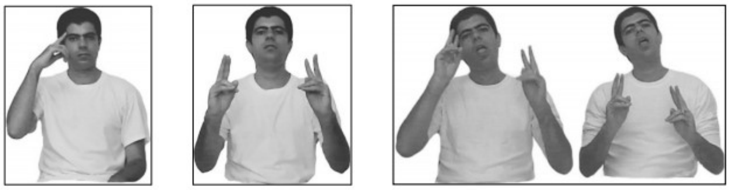
Figure 8: ISL FST for ASL Partial reduplication

These results again find correspondence Chandlee (2014), who uses an example of final-syllable reduplication in Marshallese to show the Strictly k-Local nature of bounded partial reduplication patterns. Full Reduplication, on the other hand, a widespread process in sign language, is unbounded, requiring the full power of the regular languages (though see Dolatian & Heinz (2017) for alternatives). This is an interesting case for future work.

4.3 Compound Reduction

Many lexicalized sign compounds undergo a type of phonological reduction (Frishberg 1975). First of all, some sequential segments of both members of the compound delete (Liddell 1984; Liddell & Johnson 1989). In addition, the hand configuration of the first member also deletes, and that of the second member characterizes the whole surface compound (Sandler 1986, 1989).

a. MIND b. DROP c. FAINT (ASL)



a. MIND b. STOP (suspend) c. MIND^STOP=DAYDREAM

a. THINK b. MARRY c. BELIEVE

Figure 9: Compound Reduction in ASL. Top: MIND+DROP=FAINT; Middle: MIND+STOP=DAYDREAM; Bottom: THINK+MARRY=BELIEVE. (from (Sandler & Lillo-Martin 2006))

The compound THINK+MARRY=BELIEVE is an example, illustrated in Figure 9. What is significant in this reduction process is that the hand configuration is not lined up temporally with the locations and movements. Rather, one segment of the first member of the compound survives, but its original hand configuration deletes, and the hand configuration of the second member spreads to characterize it. This is an example of autosegmental stability, a defining property of autosegments. Figure 10 is a Hand Tier representation of the compound reduction process.

Once again, the finite nature of the sign enables construction of a FST to capture this process. To meet the requirements of strict locality, though, it must be shown that there is some finite k-value in the input that directs the transduction to the

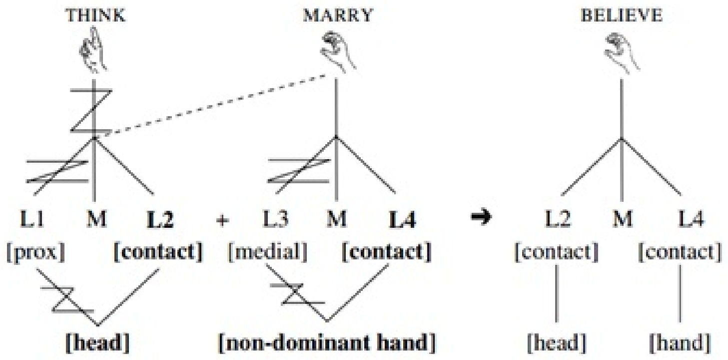


Figure 10: Compound Reduction for ASL THINK+MARRY=BELIEVE (from (Sandler & Lillo-Martin 2006)).

output. Here again boundedness of the sign helps. As compounding only involves two input signs (they may be mono- or bisyllabic), each reduction process only has to look back a finite number of segments. In the case of BELIEVE, above, there are 6 segments to see. I again use the may assume a string representation for BELIEVE as follows:

(7) $\rtimes L_1^1 M^1 L_2^1 \ltimes \rtimes L_3^2 M^2 L_4^2 \ltimes \rightarrow L_2^2 M^2 L_4^2$

In (7), the string representation shows a sequence of Ls and Ms, augmented with word-edge symbols ($\ltimes \rtimes$). Since each handshape characterizes each segment within a single sign, and each place feature characterizes a segment, they are represented with superscripts and subscripts, respectively.

Analysis of the compounding is in two steps. First, starting from the right edge, the HC of the second sign (noted by superscript 2 in (7)) spreads to characterize the first sign as well. As this process only depends on the previous $k - 1$ symbol in the input, it is a ISL-2. Second, deletion processes eliminate each onset Ls and the first M. It is tempting to analyze this deletion as a spreading operation, since the output sign agrees in place and manner features. However, compounding of syllables whose coda fetaures are different preserves these features in reduction. However, deletion is dependent on a finite number of segments in the input, and thus is also ISL-k for some k, just as the spreading operation would be. Though the k-size may be different, it is a minuscule amount and does not bias the analysis against a deletion account.

5 Algebraic Phonology, Amodality, and Cognition

The preceding section asked whether certain sign language processes require the same class of complex functions posited for processes in spoken language. The answer immediately turned out to be affirmative, given the particular nature of sign syllables. In particular the bounded sequential aspect of a sign syllable constrains the operations on its input segments so that they obey a condition of Input Strict Locality. For two of the processes (partial reduplication and contact metathesis), the spoken equivalent shows exactly the same level of complexity. Additionally, the sign-specific process of compound reduction also fell into the class. ISL mappings for particular k-values seem to subserve phonological processes regardless of the modality they sit in, at least for some.

A result like this supports the view that at least some part of phonological computation is amodal, algebraic, and autonomous. Further research in this direction has particularly interesting consequences: if phonology is truly independent of modality, then any phonological processes will fit into this subregular characterization regardless of modality. If not, then either (1) the subregular hierarchy is not expressive enough, (2) the signed modality is not as expressive as the oral modality, or (3) the "algebraic" view is wrong.

Initially, this result offers a crystal clear computational formulation of Berent (2013)'s claim that universal primitives and constraints might be shared cross-modally. Cross-modal computation of this sort challenges the view that an algebraic phonological grammar can emerge solely from non-algebraic systems, such as the phonetic component, and suggests that phonological principles are irreducible to phonetic pressures. Some phonetically plausible restrictions are unattested phonologically, whereas other phonologically attested restrictions are not phonetically optimal. The demonstrable dissociation between phonological constraints and their phonetic basis, on the one hand, and the commonalities across modalities, on the other, suggests that the design of the phonological grammar is shaped by common universal principles that are distinct from phonetic knowledge.

But let's be careful. Although these observations are consistent with the hypothesis of a specialized core system for phonology, they may be criticized as being simultaneously both too weak and too strong a test for the hypothesis. On the one hand, the search for phonetically arbitrary principles of phonological organization might be unnecessarily restrictive. Although the detection of such arbitrary principles would certainly implicate an abstract phonological cause, there is no reason to expect that had a core phonological system existed, its design should have been divorced from the properties of speech – the default medium of language in all hearing communities. In fact, some variation across modalities is expected. If grammatical constraints are the phenotypic expression of innate phonological knowledge, then variation in triggering conditions (e.g., input modality) is bound to yield variations in the resulting phonological system. So while the subregularity of cross-modal phonological processes is certainly significant, the underlying expectation that core phonological knowledge be amodal presents too strong a test of the subregular hypothesis.

To address this shortcoming, we must combine findings from typology and formal analysis with experimental results on the effect of these subregular constraints on the behavior of individual speakers. As stated above, these classes have well-defined cognitive parallels conveying information about grammaticality that phonological competence needs to be mindful of when processing strings (**?**; Rogers *et al.* 2013). Additionally, the lower, simpler classes in the hierarchy have more efficient learning algorithms (Heinz *et al.* 2012; Jardine & McMullin 2016) and appear to be more easily learnable by humans (Lai 2015; Hwangbo 2015; Avcu 2017).

In addition to its cognitive plausibility for phonology, the subregular hypothesis has very recently been extended to other areas of linguistic knowledge: morphotactics/morphology (Aksënova *et al.* 2016; Chandlee 2017), morphosemantics (De Santo & Graf 2017) and even syntax, if the data structure are viewed as derivation trees rather than strings (Graf & Heinz 2016). This allows for a strongly testable avenue about modality issues in general. The fruitfulness of this approach both in these areas and now to issues of phonological modality opens the door for a completely new linguistic modalities and a completely new characterization of linguistic competence.

6 Conclusion

I analyzed three processes in sign phonology using particular finite-state machines to argue that phonological processes are amodal and inhabit the same complexity class as their spoken counterparts. This work points to an autonomous phonological competence and highlights the usefulness of formal language theory in illuminating modality issues.

7 Acknowledgments

I would like to thank Thomas Graf, Jeff Heinz, and Mark Aronoff for their continual expertise and help, and members of the Computational Linguistics Lab at Stony Brook for their helpful advice.

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